

SAFETY ELLIPSE MOTION WITH COARSE SUN ANGLE OPTIMIZATION

Bo J. Naasz
NASA Goddard Space Flight Center

ABSTRACT

The Hubble Space Telescope Robotic Servicing and De-orbit Mission (HRSDM) was to be performed by the unmanned Hubble Robotic Vehicle (HRV) consisting of a Deorbit Module (DM), responsible for the ultimate disposal of Hubble Space Telescope (HST) at the end of science operations, and an Ejection Module (EM), responsible for robotically servicing the HST to extend its useful operational lifetime. HRSDM consisted of eight distinct phases, including: launch, pursuit, proximity operations, capture, servicing, EM jettison and disposal, science operations, and deorbit. The scope of this paper is limited to the Proximity Operations phase of HRSDM. It introduces a relative motion strategy useful for Autonomous Rendezvous and Docking (AR&D) or Formation Flying missions where safe circumnavigation trajectories, or close proximity operations (tens or hundreds of meters) are required for extended periods of time.

Parameters and algorithms used to model the relative motion of HRV with respect to HST during the Proximity Operations phase of the HRSDM are described. Specifically, the Safety Ellipse (SE) concept, convenient parameters for describing SE motion, and a concept for initializing SE motion around a target vehicle to coarsely optimize sun and relative navigation sensor angles are presented. The effects of solar incidence angle variations on sun angle optimization, and the effects of orbital perturbations and navigation uncertainty on long term SE motion are discussed.

INTRODUCTION

The Proximity Operations (ProxOps) phase of an AR&D mission to a non-cooperative target* such as a non-functional HST is likely to include a period of target surveillance, where the chase vehicle points its cameras, and relative navigation sensors at the target vehicle for an extended amount of time in order to characterize the target's configuration, and allow relative orbit and attitude estimation processes to converge. Whereas manned rendezvous and docking missions and AR&D missions to cooperative targets can perform these tasks rather quickly (time frames on the order of tens of minutes), missions such as the HRSDM could require significantly more time, and more complicated relative motion to fully plan the AR&D sequence.

In the HRSDM ProxOps phase, the HRV must achieve a relative trajectory with respect to a potentially non-functional HST that allows it to characterize the telescope's configuration, including positioning of the solar arrays, high gain antennas, and aperture door. Additionally, the HRV sensor suite must be pointed at the telescope for an uncertain amount of time to allow relative attitude and translation estimation processes to converge. The relative motion trajectory must: 1) allow for operations on the order of hours or days; 2) provide multiple view angles to improve the inspection capability; 3) provide

*Non-cooperative refers to a target that was not designed to be autonomously docked with, or is incapable of providing critical data to the chase vehicle. HST can be considered non-cooperative because it does not provide relative orbit and attitude estimation information; also, a non-functional HST could potentially be tumbling, and therefor not in a cooperative orientation.

safe, passive collision avoidance; and 4) satisfy the requirements of the vehicle's thermal, power, and communications subsystems.

To satisfy these relative motion requirements, the HRSDM flight dynamics team has expanded on a relative motion concept briefly suggested by Wigbert Fehse in [3]. The relative motion trajectory follows an elliptical path around the target vehicle, with out-of-plane motion timed such that the relative motion trajectory never crosses the velocity vector of the target vehicle. In this relative trajectory, drift of the two spacecraft (for example due to relative state estimation error) will not result in re-contact, and the trajectory can be considered passively safe. Using Fehse's terminology, we refer to this safe relative motion as a "safety ellipse".

In this paper we develop an approximation of spacecraft relative motion to assist in the design of a relative trajectory that suits the needs of HRV in the Proximity Operations phase to rendezvous with, inspect, and ultimately capture HST while maintaining a safe, collision-free trajectory and optimizing lighting conditions for solar power collection and relative navigation.

SYMBOLS

a	reference orbit semi-major axis
α	angle between the mean inertial relative position direction and the radial direction
β	orbit-plane solar-incidence angle
χ	out-of-plane phase of relative motion
γ	in-plane phase of relative motion
μ	Earth gravitation constant
n	mean motion
ϕ	angular position in the in-plane two-by-one ellipse
ψ	difference between in-plane and out-of-plane phase of relative motion
ρ	spacecraft to spacecraft relative position vector
$\bar{\rho}$	mean inertial relative position vector
\hat{S}_i	inertial Sun direction
x, y, z	Hill's frame components of the vector ρ
x_{max}	magnitude of radial periodic motion
y_c	mean offset of periodic relative motion in the velocity direction
z_{max}	magnitude of out-of-plane periodic motion
ζ	angular deviation of relative motion from the mean inertial direction
ζ_{IP}	in-plane component of ζ
ζ_{OOP}	out-of-plane component of ζ

COORDINATE FRAMES

The standard Earth-centered inertial frame is designated as \mathcal{F}_i and has unit vectors $\hat{\mathbf{i}}_1, \hat{\mathbf{i}}_2$ and $\hat{\mathbf{i}}_3$ with $\hat{\mathbf{i}}_1$ in the direction of vernal equinox, $\hat{\mathbf{i}}_3$ in the direction of the Earth spin axis, and $\hat{\mathbf{i}}_2 = \hat{\mathbf{i}}_3 \times \hat{\mathbf{i}}_1$. Frame \mathcal{F}_i has its origin at the center of Earth.

The LVLH frame, \mathcal{F}_a , has unit vectors $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2$ and $\hat{\mathbf{a}}_3$ with $\hat{\mathbf{a}}_3$ in the nadir direction, $\hat{\mathbf{a}}_2$ in the direction of the negative orbit normal, and $\hat{\mathbf{a}}_1 = \hat{\mathbf{a}}_2 \times \hat{\mathbf{a}}_3$, which is the velocity direction for a circular orbit. Frame \mathcal{F}_a has its origin at the reference spacecraft to which it is associated.

The orbital frame, \mathcal{F}_o , is a permutation of the LVLH frame, and has unit vectors $\hat{\mathbf{o}}_1$ in the radial direction, $\hat{\mathbf{o}}_2$ in the co-velocity direction and $\hat{\mathbf{o}}_3$ in the orbit normal direction. Frame \mathcal{F}_o has its origin

at the reference spacecraft to which it is associated. The orbital frame is also commonly referred to as Hill's frame, or the RIC (radial, in-track, cross-track) frame.

The HRV body frame, \mathcal{F}_H , has unit vectors $\hat{\mathbf{H}}_1$, $\hat{\mathbf{H}}_2$ and $\hat{\mathbf{H}}_3$ with $\hat{\mathbf{H}}_1$ in the relative navigation sensor boresight direction, and parallel to the HST boresight direction when mated, $\hat{\mathbf{H}}_3$ normal to the DM solar panels, and normal to the HST solar panels when mated, and $\hat{\mathbf{H}}_2 = \hat{\mathbf{H}}_3 \times \hat{\mathbf{H}}_1$.

RELATIVE ORBITAL DYNAMICS REVIEW

The relative motion of a spacecraft with respect to a reference spacecraft in a circular or nearly circular orbit can be approximated using linearized equations of motion. These linear equations of motion, known as the Hill-Clohesy-Wiltshire equations,¹ are commonly expressed in a coordinate frame centered at the reference spacecraft, and rotating with the orbital rate. This frame, often referred to as Hill's frame, is identical to frame \mathcal{F}_o , with unit vectors $\hat{\mathbf{o}}_1$ in the radial direction, $\hat{\mathbf{o}}_2$ in the co-velocity direction and $\hat{\mathbf{o}}_3$ in the orbit normal direction. The variables x , y , and z are often used to describe the magnitudes of the relative position vector, ρ in Hill's frame as given in Eq. (1).

$$\rho = x\hat{\mathbf{o}}_1 + y\hat{\mathbf{o}}_2 + z\hat{\mathbf{o}}_3 \quad (1)$$

The linearized Hill's equations given in Eqs. (2-4) approximate the relative motion of two spacecraft in an unperturbed point-mass gravitational field.

$$\ddot{x} - 2n\dot{y} - 3n^2x = 0 \quad (2)$$

$$\ddot{y} + 2n\dot{x} = 0 \quad (3)$$

$$\ddot{z} + n^2z = 0 \quad (4)$$

The mean motion, n , of the reference spacecraft is given in terms of the central body gravitation parameter, μ , and the reference spacecraft semi-major axis, a , by the equation:

$$n = \sqrt{\mu/a^3} \quad (5)$$

The linearized equations of motion can be solved analytically. The solution is:

$$x(t) = (\dot{x}_0/n) \sin nt - (3x_0 + 2\dot{y}_0/n) \cos nt + 4x_0 + 2\dot{y}_0/n \quad (6)$$

$$y(t) = (2\dot{x}_0/n) \cos nt + (6x_0 + 4\dot{y}_0/n) \sin nt - (6nx_0 + 3\dot{y}_0)t - 2\dot{x}_0/n + y_0 \quad (7)$$

$$z(t) = (\dot{z}_0/n) \sin nt + z_0 \cos nt \quad (8)$$

This solution is often used to assist in the design of spacecraft relative motion trajectories. For example, to build a relative motion that does not include secular growth of the spacecraft relative range, we remove the secular term in Eq. (7) by enforcing the following constraint on the radial position and in-track velocity:

$$\dot{y}_0 = -2x_0n \quad (9)$$

As described in [2], this constraint results in an orbit displaced from the reference orbit, but with the same energy, or semi-major axis, and thus the same orbital period (neglecting higher order terms in eccentricity, and the effect of Earth oblateness). Rewriting Hill's equation with this period matching constraint, the relative equations of motion become:

$$x(t) = (\dot{x}_0/n) \sin nt + x_0 \cos nt \quad (10)$$

$$y(t) = (2\dot{x}_0/n) \cos nt - 2x_0 \sin nt - 2\dot{x}_0/n + y_0 \quad (11)$$

$$z(t) = (\dot{z}_0/n) \sin nt + z_0 \cos nt \quad (12)$$

These equations describe a relative motion in which the in-plane and out-of-plane motions are decoupled. Furthermore, the in-plane motion can be classified into three categories: 1) constant offset in the velocity direction; 2) elliptical motion in the radial, in-track plane, with period equal to the orbital period, and with major axis in the velocity direction twice the magnitude of the minor axis in the radial direction; 3) combinations of 1) and 2). The out-of-plane motion is a simple harmonic oscillator with period equal to the orbital period. Using these well known equations describing spacecraft relative motion, we now introduce the safety ellipse concept, and a new parameterization and equations of relative motion for two vehicles in close-proximity, near-circular orbits.

SAFETY ELLIPSE DEFINITION

To further improve our understanding of relative motion, we introduce six new independent variables to describe the relative motion: the minor axis of the in-plane ellipse, or the maximum radial offset, x_{max} ; the magnitude of the out-of-plane motion, z_{max} ; the instantaneous mean offset of relative motion in the velocity direction, y_c ; the drift rate of the mean offset in the velocity direction, \dot{y}_c ; the phase angle of the radial motion, γ (with zero phase defined as the point where the radial offset is maximized and radial velocity is zero), the phase angle of the out-of-plane motion, χ , (with zero phase defined as the point where the out-of-plane offset is maximized and out-of-plane velocity is zero). These independent variables are shown in Figs. 1 and 2. Note that γ and χ both have rates given by the mean motion,

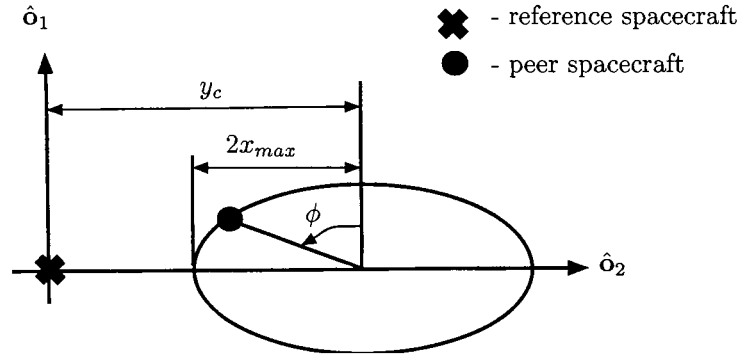


Figure 1 In-plane relative spacecraft motion in Hill's frame

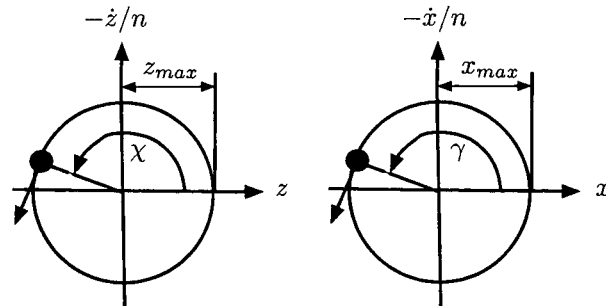


Figure 2 Variables describing relative in-plane and out-of-plane phase

and can be written as functions of time, as follows:

$$\gamma(t) = nt - nt_{\gamma=0} \quad (13)$$

$$\chi(t) = nt - nt_{\chi=0} \quad (14)$$

where $t_{\gamma=0}$ is the time of max radial offset, and $t_{\chi=0}$ is the time of max out-of-plane offset. It is convenient to define a new variable, ψ , the difference between the initial in-plane and out-of-plane phase angles as follows:

$$\psi = \chi - \gamma \quad (15)$$

Assuming the period matching constraint has been met, γ and χ can also be written as

$$\gamma = \arctan\left(\frac{-\dot{x}/n}{x}\right) \quad (16)$$

$$\chi = \arctan\left(\frac{-\dot{z}/n}{z}\right) \quad (17)$$

If period matching is not met, the secular drift can be described in terms of \dot{y}_c , which is defined in Eq. (18) to include the difference between the initial velocity and the velocity required by the period matching constraint in Eq. (9),

$$\dot{y}_c \equiv \dot{y} + 2x_{max}n \cos \gamma \quad (18)$$

Assuming $t_0 = t_{\gamma=0}$, the in-plane initial conditions include contributions from periodic motion, and secular motion from the initial value of \dot{y}_c , which can be written as:

$$x_0 = x_{max} + f(\dot{y}_c) \quad (19)$$

$$y_0 = y_{c0} + \dot{y}_c t \quad (20)$$

To define the function $f(\dot{y}_c)$ in Eq. (19), assume relative motion with secular drift only, such as in a relative motion with semi-major axis difference only. Rewriting Eq. (11) with the appropriate initial conditions ($x_{max} = 0$, $\dot{y} = \dot{y}_c$, $\dot{x} = 0$), the $y(t)$ equation becomes:

$$y(t) = (6x_0 + 4\dot{y}_c/n) \sin(nt) - (6nx_0 + 3\dot{y}_c)t \quad (21)$$

Requiring the periodic component of this equation to equal zero results in the following definition:

$$f(\dot{y}_c) = -\frac{2\dot{y}_c}{3n} \quad (22)$$

Rewriting Eqs. (10-12) with the initial conditions

$$t_0 = t_{\gamma=0} \quad (23)$$

$$x_0 = x_{max} - \frac{2\dot{y}_c}{3n} \quad (24)$$

$$y_0 = y_c \quad (25)$$

$$\dot{x}_0 = 0 \quad (26)$$

$$\dot{y}_0 = \dot{y}_c - 2x_{max}n \quad (27)$$

$$(28)$$

we get an expression for the relative motion in terms of the six new independent parameters:

$$x(\gamma) = x_{max} \cos \gamma - \frac{2\dot{y}_c}{3n} \quad (29)$$

$$y(\gamma) = -2x_{max} \sin \gamma + \dot{y}_c \left(\frac{\gamma}{n}\right) + y_c \quad (30)$$

$$z(\gamma) = z_{max} \cos(\gamma + \psi) \quad (31)$$

We can also write the time derivatives of the relative position states as follows:

$$\dot{x}(\gamma) = -x_{max}n \sin \gamma \quad (32)$$

$$\dot{y}(\gamma) = -2x_{max}n \cos \gamma + \dot{y}_c \quad (33)$$

$$\dot{z}(\gamma) = -z_{max}n \sin(\gamma + \psi) \quad (34)$$

The reverse operations to compute the new parameters from relative position and velocity is fairly straightforward, and are given as follows:

$$\gamma = \arctan\left(\frac{-\dot{x}/n}{x}\right) \quad (35)$$

$$\chi = \arctan\left(\frac{-\dot{z}/n}{z}\right) \quad (36)$$

$$x_{max} = \left| \frac{x}{\cos \gamma} \right| \quad (37)$$

$$z_{max} = \left| \frac{z}{\cos \chi} \right| \quad (38)$$

$$\dot{y}_c = \dot{y} + 2x_{max}n \cos \gamma \quad (39)$$

$$y_c = y + 2x_{max} \sin \gamma - \dot{y}_c \left(\frac{\gamma}{n}\right) \quad (40)$$

It will be convenient later to define another variable, ϕ , as the angular position in the two-by-one in-plane ellipse, measured counter-clockwise (in the direction of relative motion) from the radial direction to the current in-plane position of the reference spacecraft. Note that ϕ and γ are only equal when $x = 0$, or $x = x_{max}$, where their values are 0 , $\pi/2$, π , etc. The in-plane angular position, ϕ , can be written as a function of time for the period-matched motion using Eqs. (10) and (11) as follows:

$$\phi(t) = \arctan(-y(t)/x(t)) \quad (41)$$

$$\phi(t) = \arctan\left[\frac{-(2\dot{x}_0/n \cos(nt) - 2x_0 \sin(nt) - 2\dot{x}_0/n)}{\dot{x}_0/n \sin(nt) + x_0 \cos(nt)}\right] \quad (42)$$

Defining initial conditions as in Eqs. (24-27), $\phi(t)$ can be written as

$$\phi(t) = \arctan\left[\frac{2 \sin(nt)}{\cos(nt)}\right] \quad (43)$$

The magnitude and phase of the out-of-plane motion are independent of the in-plane variables. Varying the difference between the initial in-plane and out-of-plane angles (henceforth, the phase difference, or ψ), the relative motion changes as shown in Fig. 3. This figure illustrates how the phase difference is what truly impacts the orientation of the plane of relative motion, and is a more appropriate choice for describing the relative motion than either γ or χ . In fact, γ and χ vary continuously over the orbit, making them difficult parameters to use in relative motion design. As shown in the figure, at zero degrees phase difference, the relative motion is restricted to planes defined by rotating the \hat{o}_1 - \hat{o}_2 plane about the co-velocity \hat{o}_2 direction. Some examples of relative motion with zero degrees phase difference are the Sabol-Burns-McLaughlin (SBM)-Circular formation, in which the relative motion follows a circular trajectory, and the SBM-Projected-Circular formation, which projects a circle in the \hat{o}_2 - \hat{o}_3 plane.²

At 90 degrees phase difference, the relative motion is restricted to planes defined by rotating the \hat{o}_1 - \hat{o}_2 plane about the radial (\hat{o}_1) direction. One example of relative motion with 90 degrees phase difference is another projected-circular formation, which projects a circle into the radial-cross track (\hat{o}_1 - \hat{o}_3) plane. Relative motion trajectories with 90 degrees phase difference have a particularly useful quality, which

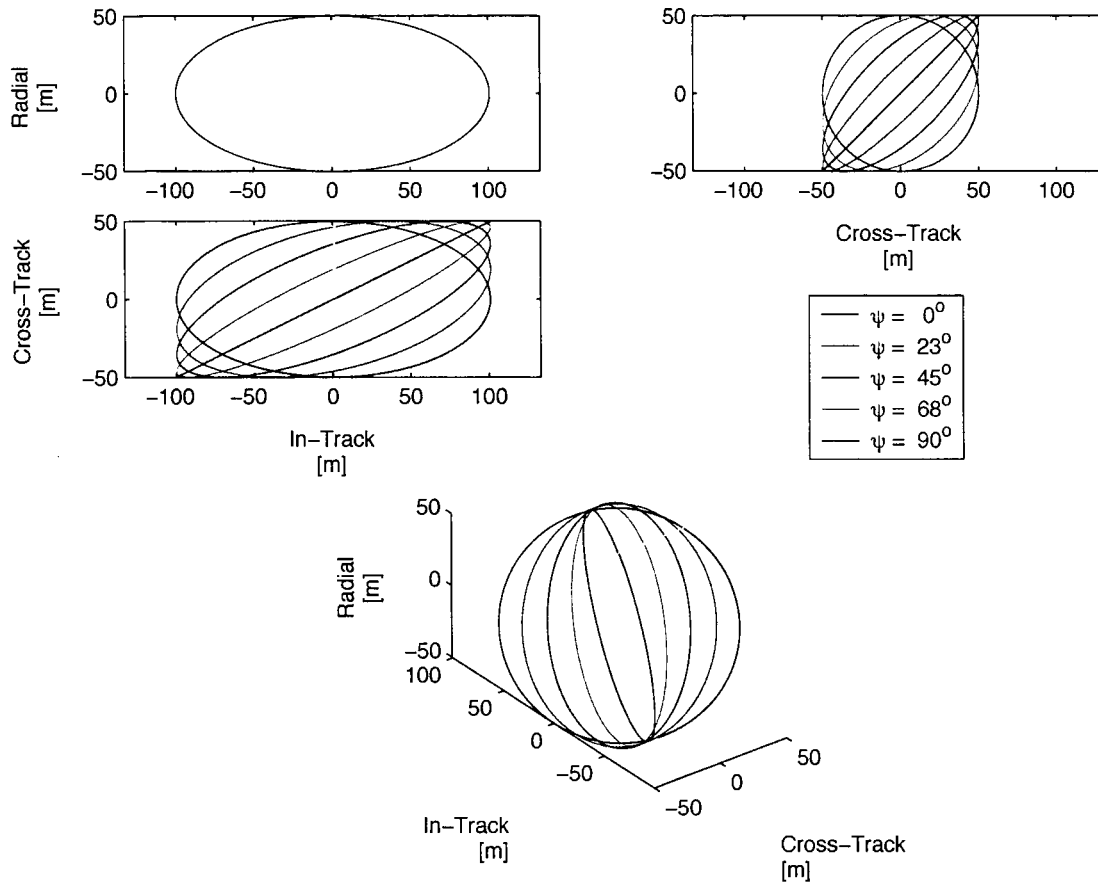


Figure 3 Relative motion trajectories with varying phase difference between in-plane and out-of-plane motion, with $y_c = 0$, $x_{max} = 50\text{m}$, $z_{max} = 50\text{m}$. The circular motion in the top right plot corresponds to $\psi = 90^\circ$, the requirement for safety ellipse motion.

is that the relative motion trajectory never crosses the velocity direction of the reference spacecraft. In this trajectory, if the in-plane and out-of-plane separation magnitude is large enough, relative drift of the two spacecraft (for example due to relative state estimation error) will not result in re-contact. The peer spacecraft trajectory will remain on the surface of a circular cylinder, with axis parallel to the reference spacecraft velocity direction, as illustrated in Fig. 4. This relative motion concept is briefly suggested by Wigbert Fehse in [3]. Using Fehse’s terminology for the motion, we refer to any relative motion with 90 degrees phase difference as a “safety ellipse”. Safety ellipse motion of two spacecraft that do not satisfy the period matching constraint in Eq. (9) will henceforth be referred to as a “walking safety ellipse”. Note that a walking safety ellipse requires a sixth independent variable to fully describe the relative motion. A convenient choice for this variable is the rate of change of the mean offset in the velocity direction, or \dot{y}_c . Another convenient choice would be the mean radial offset of the elliptical motion.

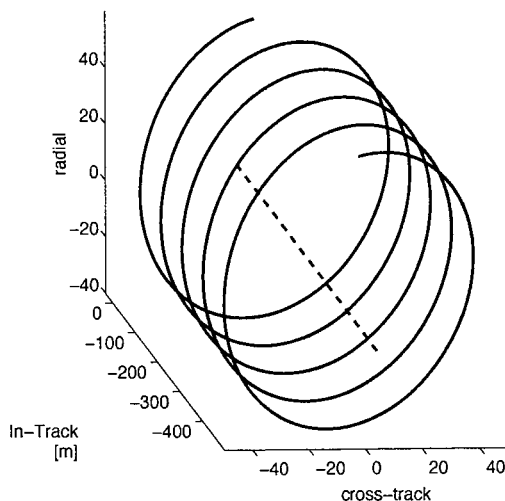


Figure 4 Relative motion in the walking safety ellipse, with drift rate $\dot{y}_c = 50$ meters per orbit

POWER AND LIGHTING CONSIDERATIONS

The desired HRV attitude during most of the ProxOps phase is most strictly constrained by HRV $\hat{\mathbf{H}}_1$ -axis (the relative navigation sensor axis) pointing at HST to enable relative navigation. The other primary constraint on the HRV attitude is solar power collection, which would optimally point either the HRV $\hat{\mathbf{H}}_3$ -axis (DM solar panel axis) or the HRV negative $\hat{\mathbf{H}}_1$ -axis (EM solar panel axis) directly towards the sun. The relative navigation pointing and sun pointing constraints cannot be satisfied simultaneously, as there is no situation in which vectors from HRV to HST and from HRV to Sun remain perpendicular (for $\hat{\mathbf{H}}_3$ -axis sun point) or antiparallel (for minus $\hat{\mathbf{H}}_1$ -axis sun point) throughout the orbit. The sun-pointing mode carries an inertial pointing constraint, while the HST-pointing mode carries an attitude tracking constraint. Assuming $\hat{\mathbf{H}}_1$ -axis HST-pointing is the primary constraint, the best we can do is to maintain $\hat{\mathbf{H}}_1$ -axis pointing at HST, and maximize the solar power generation by careful selection of relative motion initial conditions. In this section we introduce an algorithm for initializing the safety ellipse motion which enables $\hat{\mathbf{H}}_1$ -axis HST-pointing, while maximizing solar power collection on either the DM or EM solar panels.

Safety Ellipse Relative Motion in Inertial Coordinates

Inspection of the relative motion of two spacecraft in a safety ellipse configuration yields a useful observation: assuming safety ellipse parameters $y_c = 0$, $x_{max} = 50$ meters, and $z_{max} = 50$ meters, the inertial direction from one spacecraft to the other varies by no more than about 30 degrees from some mean inertial direction. Furthermore, this mean inertial direction lies in the plane of the orbit, and can be steered within the plane of the orbit by careful selection of the initial safety ellipse in-plane phasing, γ_0 .

The concept of a mean inertial direction from a spacecraft positioned in a safety ellipse to another, henceforth referred to as the vector $\bar{\rho}_i$, is illustrated in Figures 5 and 6. Figure 5 shows the relative motion of two spacecraft in an inertial frame centered at one of the spacecraft[†]. In the figure, the reference spacecraft has zero inclination, so that motion in the \hat{i}_3 -axis corresponds to out-of-plane motion. Note that the relative motion traces out a skewed figure-eight path which is entirely within a 30-degree-half-angle cone. The cone axis of symmetry is aligned with the center of the figure-eight path at the point where the trajectory crosses over itself. This axis of symmetry is also the vector describing the mean relative position vector, $\bar{\rho}_i$.

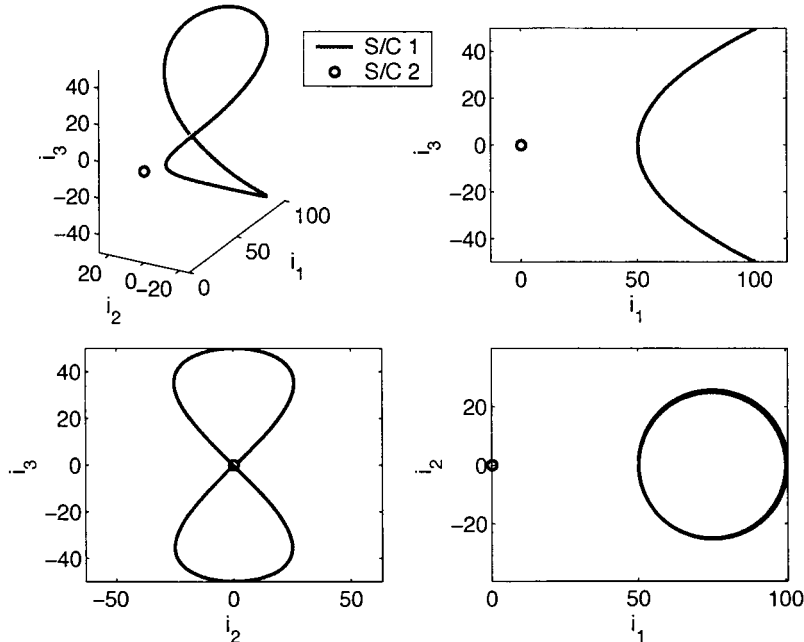


Figure 5 Safety ellipse relative motion in inertial coordinates. Reference spacecraft has zero inclination, so \hat{i}_3 -axis corresponds to out-of-plane motion in this figure. Safety ellipse parameters are: offset, $y_c = 0$; in-plane magnitude, $x_{max} = 50$ meters; out-of-plane magnitude, $z_{max} = 50$ meters.

Figure 6 shows the inertial relative position of 5 spacecraft positioned in a safety ellipse, with respect to a reference spacecraft. The initial in-plane phase angles, γ_0 , of the five spacecraft are incremented to fill the 360 degree space. The figure illustrates how $\bar{\rho}_i$ varies as a function of γ_0 . Note that the mean relative position vector is always parallel to the orbital plane, and the instantaneous relative position never deviates from the mean direction by more than 30 degrees for a safety ellipse with parameters described above.

[†]It matters not which spacecraft is in the center (the reference spacecraft) in Figure 5, as the inertial relative position of one with respect to the other involves only a sign change.

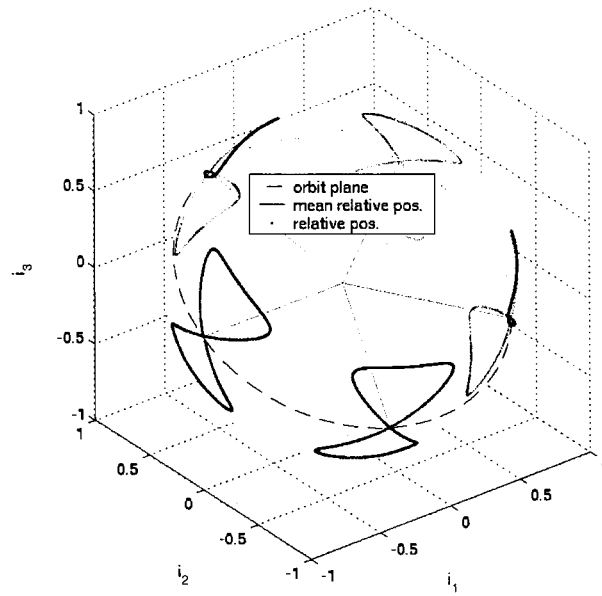


Figure 6 Safety ellipse mean inertial direction for varying initial in-plane phase, γ_0 . Reference spacecraft has HST orbital parameters. Safety ellipse parameters are: offset, $y_c = 0$; max radial magnitude, $x_{max} = 50$ meters; out-of-plane magnitude, $z_{max} = 50$ meters.

By carefully selecting γ_0 at the initialization of safety ellipse motion, we can steer the mean inertial direction to any orientation in the plane of the orbit. This is particularly useful in our quest to optimize solar incidence angles while enforcing strict pointing of a relative navigation suite at a target spacecraft. We previously described the attitude constraints on HRV as a strict pointing of $\hat{\mathbf{H}}_1$ -axis at HST, and a less strict but extremely important sun pointing requirement, defined by either $\hat{\mathbf{H}}_3$ -axis sun pointing, or minus $\hat{\mathbf{H}}_1$ -axis sun pointing. We can coarsely optimize the safety ellipse motion for solar power collection on the DM $\hat{\mathbf{H}}_3$ panel by forcing the mean inertial direction from HRV to HST, $\bar{\rho}_i$, to be oriented as close to perpendicular to the inertial sun direction, $\hat{\mathbf{S}}_i$ as possible. This concept is illustrated in Figure 7. Likewise, we can coarsely optimize solar collection on the EM $\hat{\mathbf{H}}_1$ panel by forcing $\bar{\rho}_i$ to be antiparallel to the HST-sun direction.

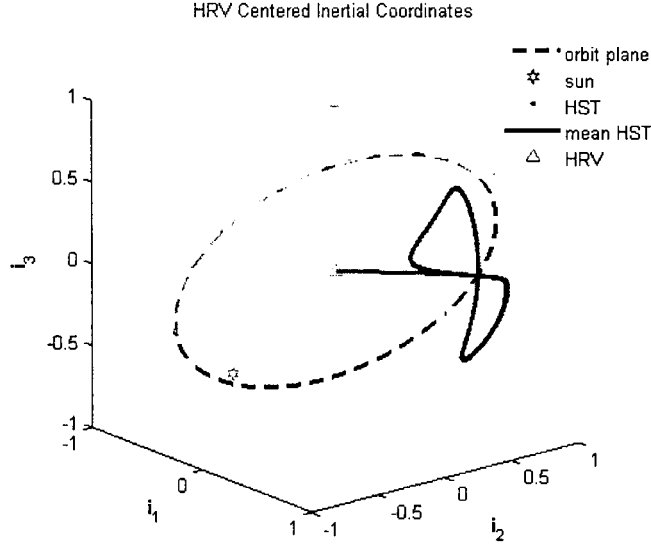


Figure 7 Safety ellipse motion with initial in-plane phase, γ_0 , coarsely optimized for solar power collection by forcing the mean relative direction to HST to be perpendicular to the sun vector.

The problem of coarsely optimizing the safety ellipse motion of HRV about HST can be expressed as follows: given HST initial conditions at time t_1 , find the initial in-plane phase $0 < \gamma_1 < 2\pi$ and the initial phase difference between in-plane and out-of-plane motion, $\psi_1 = \pi/2$ or $3\pi/2$, such that the mean inertial direction from HRV to HST, $\bar{\rho}_i$, is either: a) perpendicular to the sun vector, for DM solar array pointing; or b) antiparallel to the sun vector, for EM solar array pointing.

To optimize the placement of $\bar{\rho}_i$, we also need to define an angle which describes the orientation of the mean inertial direction as a function of time. We will call this angle α , and define it as the angle between the reference spacecraft radial direction, $\hat{\mathbf{o}}_1$, and the desired mean inertial direction expressed in the orbital frame $\bar{\rho}_o$, with $\alpha = 0$ at the same point where $\gamma = 0$, and measured positive in the clockwise direction; α can be written as a function of $\bar{\rho}_o$, as follows:

$$\alpha = \arctan \left(\frac{-\bar{\rho}_{o2}}{\bar{\rho}_{o1}} \right) \quad (44)$$

Since $\bar{\rho}$ is an inertially fixed vector[†] parallel to the orbit plane, and since the reference HST orbit is nearly circular, the time derivative of α is approximately equal to the orbital rate or mean motion, n .

Eq. (44) gives us the initial value of γ required to align the mean inertial vector from HRV to HST in whatever direction (within the orbit plane) is required to maximize solar power collection. Note that because $\gamma \neq \phi$, the direction from HRV to HST will not be optimally aligned throughout the orbit. Combining Eqs. (13) and (43), we can determine the in-plane deviation from the mean direction to be ζ_{IP} as given in Eq. (45). Note that the maximum in-plane deviation is approximately 19.41° .

$$\zeta_{IP} = \phi - \gamma = \arctan\left(\frac{2 \sin \gamma}{\cos \gamma}\right) - \gamma \quad (45)$$

The maximum out-of-plane deviation of the motion from the mean direction is a function of the safety ellipse parameters, and, given equal radial and out-of-plane maximum offsets, is approximately 26.56° .

$$\zeta_{OOP} = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \quad (46)$$

The total angular deviation from the mean direction for safety ellipse motion is a combination of in-plane and out-of-plane deviations, and is typically less than about 30° .

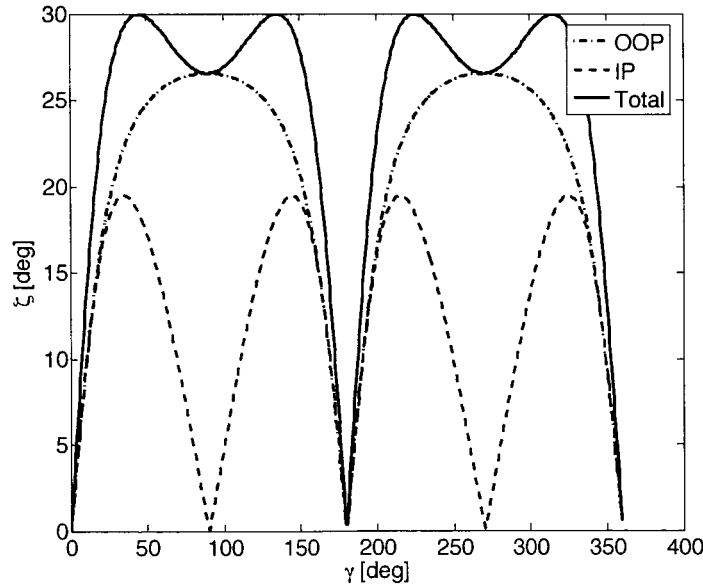


Figure 8 Angular deviation of HST motion from the mean direction, $x_{max} = z_{max} = 75\text{m}$

For the DM solar panel optimization, we choose γ such that the mean direction from HRV to HST is perpendicular to the sun line. This initial condition, along with clocking of HRV about the relative navigation boresight axis, allows us to point the solar panels within a 30 degree cone of the sun line (projected into the orbital plane) at all times for optimal sun angles. For the EM solar panel optimization, we choose γ such that the mean inertial direction is anti-parallel to the sun line. The EM solar panel optimization case is slightly less robust, as clocking about the navigation boresight axis cannot improve the solar incidence angle on the EM panels. Both cases are sensitive to sun-orbit plane angle (β angle) variations.

[†]This is true in the short term, but relative orbital perturbations such as differential higher order gravitational effects will cause the mean inertial direction to wander. For now we will assume $\bar{\rho}$ is inertially fixed.

Sun–Orbit Plane Angle (β angle) Considerations

The HRV sun pointing requirement during the ProxOps phase is most likely to be a minus $\hat{\mathbf{H}}_1$ sun point mode. For this mode, the safety ellipse initial phase parameters will be set such that the mean inertial direction from HRV to HST is anti-parallel to the vector from HRV to Sun, projected into the orbital plane. For low values of the orbit plane solar incidence angle, commonly referred to as the β angle, the solar panel sun incidence angle will be similar to trends shown in Fig. 8. As β increases, the entire ζ curve in Fig. 8 shifts upward by the amount β , making epochs with high β angles unfavorable for pointing at the target vehicle for long durations from the safety ellipse.

Perturbation Considerations

Differential perturbations on the HST and HRV orbits will cause significant changes in the safety ellipse motion. In particular, differential gravity effects, primarily from Earth oblateness (the J_2 effect) will cause differential drift in both the in-plane and out-of-plane directions. This differential gravitational force, along with effects from differential atmospheric drag, and navigation uncertainty, drive the requirement for safety ellipse maintenance maneuvers, either in the form of re-centering maneuvers (to counteract the in-plane drift, controlling y_c and \dot{y}_c), or re-initialization maneuvers (to counteract both the in-plane drift, and the out-of-plane drift, controlling all of the safety ellipse parameters).

CONCLUSIONS

A safe relative motion trajectory ideal for extended proximity operations of two vehicles in nearly circular orbits has been developed. While the relative motion trajectory is not new, the parameterization and mathematical description of the motion is new, and useful for further optimization of the motion to account for relative navigation and power generation requirements of an AR&D or formation flying mission. Further discussion of the effects of perturbations, and navigation uncertainty, and the frequency and scale of maintenance maneuvers are left for a future work.

ACKNOWLEDGEMENTS

Recognition is due to Rich Burns for countless hours of discussion and derivation in support of this effort.

REFERENCES

1. Clohessy, W. H. and Wiltshire, R. S., "Terminal Guidance System for Satellite Rendezvous," *Journal of the Aerospace Sciences*, Vol. 27, No. 5, 1960, pp. 653–658, 674.
2. Sabol, C., Burns, R., and McLaughlin, C. A., "Satellite Formation Flying Design and Evolution," *Journal of Spacecraft and Rockets*, Vol. 38, No. 2, 2001, pp. 270–278.
3. Fehse, W., *Automated Rendezvous and Docking of Spacecraft*, Cambridge University Press, Cambridge, UK, 2003.